

Heavy gravitino, naturalness, and sizable anomaly mediation

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Abstract

We consider the situation in which $m_{3/2} \sim O(100\text{TeV})$ for solving the gravitino problem and the other supersymmetry(SUSY) breaking parameters are $O(1\text{TeV})$ for the naturalness. We point out that the anomaly mediation cancels out the renormalization group contribution to the gaugino masses and the sfermion masses other than the stop masses at a scale which is called the mirage scale. The situation is similar to the mirage mediation, in which special boundary conditions for the SUSY breaking parameters are required, though for the stop masses and the up-type Higgs mass, such cancellation at the mirage scale does not happen. Despite no cancellation for the up-type Higgs mass, we show that the little hierarchy problem becomes less severe in this situation. One advantage of this situation over the mirage mediation is that the stop mixing parameter A_t can be larger and therefore, smaller stop mass is sufficient for 125 GeV Higgs. When the mirage scale is around TeV scale, the SUSY breaking parameters induced by the gravity mediation at the grand unification scale can be observed directly by the TeV scale experiments.

1 Introduction

The minimal supersymmetric (SUSY) Standard Model (MSSM) is still one of the most promising candidates as physics beyond the Standard model (SM). The MSSM can solve the gauge hierarchy problem and provide a dark matter candidate as the lightest supersymmetric particle (LSP). Moreover, the SUSY grand unified theory (GUT) is experimentally supported by the remarkable coincidence of three SM gauge coupling constants around 10^{16} GeV. However, many SUSY models suffer from a tuning problem, called the SUSY little hierarchy problem. This problem arises from a tension between naturalness which requires lightness of several SUSY particles and the Higgs mass $m_h = 125$ GeV[1, 2] which forces those to be heavy. Cosmologically, it has been pointed out that the decay of the gravitino spoils the success of the Big Bang Nucleosynthesis (BBN). This is called the gravitino problem[3, 4].

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One of the simplest solution for the gravitino problem is to assume that the gravitino decays before the BBN begins. For example, if the gravitino is heavier than 100 TeV, then the lifetime of the gravitino becomes of order 10^{-2} sec. At the time in the history of the universe, the proton-neutron ratio has not been fixed yet by freezing out the weak decay process. In the literature, the high scale SUSY breaking scenario[5], in which the scalar fermion masses are taken to be the same order of the heavy gravitino mass, has been studied because such scenario can realize the Higgs mass $m_h \sim 125$ GeV without the large stop mixing parameter A_t [6]. Such high scale SUSY breaking scenario has various advantages, for example, it has no SUSY flavor problem, no SUSY CP problem, etc. Unfortunately the fine-tuning problem on the Higgs mass becomes much worse in the scenario.

For the fine-tuning problem, it is preferable that the stop masses and the gaugino masses are of order 1 TeV. These two requirements, the gravitino mass $m_{3/2} \geq 100\text{TeV}$ and the sfermion masses $\tilde{m} \sim O(1\text{TeV})$, are not inconsistent with each other. Actually, both requirements are satisfied in the mirage mediation scenario[7, 8] in which the moduli[9] and anomaly[10] contributions to SUSY breaking parameters become comparable. One of the most important features in the mirage mediation is that the effective SUSY mediation scale can be lower because the renormalization group effects can be cancelled by the effect of the anomaly mediation. As a result, the little hierarchy problem may be solved[11, 12]. Unfortunately, in the mirage mediation, very specific boundary conditions for the SUSY breaking parameters are required. What happens if we take more generic boundary conditions for the SUSY breaking parameters? If the contribution of the anomaly mediation dominates that of the gravity mediation, then the mass squares of the right-handed slepton become negative. Therefore, we have an upper bound for the gravitino mass, which is nothing but $O(100\text{TeV})$.

In this paper, we will examine a scenario in which the gravitino mass is of order 100 TeV to solve the gravitino problem and the other SUSY breaking parameters, which are induced by the gravity mediation, are around the TeV scale to stabilize the weak scale.

Let us examine the little hierarchy problem in more detail, because it is one of the main purposes in this paper to improve the fine-tuning in the Higgs sector. In supersymmetric models, a quantum correction of the up-type Higgs squared mass $\Delta m_{H_u}^2$ strongly depends on the stop mass $m_{\tilde{t}}$:

$$\Delta m_{H_u}^2 \sim -\frac{3y_t^2}{8\pi^2}(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A_t^2) \ln \frac{\Lambda}{m_{\tilde{t}}}, \quad (1)$$

where Λ is the messenger scale and here we consider $\Lambda = 2 \times 10^{16}$ GeV. In order to realize the electroweak symmetry breaking without the fine-tuning, one can expect that $m_{\tilde{t}}$ is order of 100 GeV. On the other hand, the lightest CP-even Higgs mass m_h is also linked to the stop mass:

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \left[\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{A_t^2}{m_{\tilde{t}}^2} \left(1 - \frac{A_t^2}{12m_{\tilde{t}}^2} \right) \right]. \quad (2)$$

The Higgs mass $m_h = 125$ GeV, which is discovered by ATLAS and CMS, implies heavy stop mass such as several TeV. Therefore, it is difficult to get the realistic Higgs mass without destroying naturalness.

One of the solutions to avoid the little hierarchy problem is to move beyond the MSSM. For instance, one may add an extra singlet as in the next-to MSSM[13]. On the other hand, we can also reduce fine-tuning within the MSSM by lowering the messenger scale Λ such as the low-scale gauge mediation model[14]. One can also lower the messenger scale effectively in the case where several SUSY breaking contributions cancel the renormalization group (RG) evolution as in the TeV-scale

mirage mediation model [7, 11, 12]. Note that the large stop mass spoils the naturalness even if the messenger scale is small. The value of the $m_{\tilde{t}}$ with realizing the 125 GeV Higgs depends on the value of the A_t . It is minimized when $|A_t/m_{\tilde{t}}| = \sqrt{6}$ [15]. It is, however, difficult to realize the large A_t in the low-scale messenger models. In the TeV-scale mirage mediation, the model fixes the ratio $A_t/m_{\tilde{t}} = \sqrt{2}$ at the mirage scale, which is considered to be around the TeV scale. The gauge mediation model also fails to get the large A_t because it does not appear at the leading order. The value of A_t in these models is not sufficient to get the Higgs mass naturally.

What happens if we do not impose the specific condition $A_t/m_{\tilde{t}} = \sqrt{2}$ in the mirage mediation? To answer this question, we have to know what happens when the specific boundary conditions in the mirage mediation scenario are not imposed. This is one of our motivation for the work in this paper.

The paper proceeds as follows. In section 2, we recall that the anomaly mediation contribution can cancel the RG evolution of the gravity mediation contribution by the analytic solutions of one-loop RG equations of the MSSM. In section 3, we study what happens if the gravity mediation produces $O(1\text{TeV})$ SUSY breaking parameters while the gravitino mass is $O(100\text{TeV})$. Especially, we show that the little hierarchy problem becomes less severe like in the mirage mediation. And section 4 is for the discussion and summary.

2 Cancellation property of the anomaly mediation

It is known that the anomaly mediation[10] has the property to cancel the RG evolution of the gravity mediation. In this section, we will review this property by solving the one-loop RG equations for the SUSY breaking parameters in the MSSM.

2.1 Small Yukawa case

Let us see this cancellation property in the case where the Yukawa coupling can be neglected. The results in this subsection can be applied to all sfermion masses except stop masses and up-type Higgs mass m_{H_u} when bottom and tau Yukawa couplings can be neglected, i.e., $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle \ll 50$.

First we consider the gaugino mass M_a ($a = 1, 2, 3$). It satisfies the RG equation

$$\frac{d}{dt}M_a = \frac{1}{8\pi^2}b_a g_a^2 M_a \quad (3)$$

at one-loop level. Here the gauge coupling g_a obeys the RG equation

$$\frac{d}{dt}g_a = \frac{1}{16\pi^2}b_a g_a^3, \quad (4)$$

where $(b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)$ in the MSSM. Then the anomaly mediation solution is written as

$$M_a(\mu)|_{\text{anomaly}} = \frac{1}{16\pi^2}b_a g_a^2 m_{3/2}, \quad (5)$$

where $m_{3/2}$ is the gravitino mass. There is also the gravity mediation solution as follows:

$$M_a(\mu)|_{\text{gravity}} = \tilde{M}_a + \frac{1}{8\pi^2}b_a g_a^2 \tilde{M}_a \ln \frac{\mu}{\Lambda}, \quad (6)$$

where \tilde{M}_a is the mass from the gravity mediation at the cutoff scale Λ . Note that, in this paper, “the gravity mediation” does not include the anomaly mediation. Hereafter we assume that \tilde{M}_a is universal as

$$\tilde{M}_1 = \tilde{M}_2 = \tilde{M}_3 = M_{1/2}, \quad (7)$$

which is imposed if the GUT is assumed at the cutoff scale Λ . One can easily check that these two expressions satisfy the RG equation (3), respectively. These two contributions can coexist because the sum $M_a|_{\text{anomaly}} + M_a|_{\text{gravity}}$ also satisfies the same RG equation. It can be rewritten as

$$M_a(\mu) = M_{1/2} + \frac{1}{8\pi^2} b_a g_a^2 M_{1/2} \ln \frac{\mu}{M_{\text{mir}}}, \quad (8)$$

where the mirage scale M_{mir} is defined as

$$\ln \frac{M_{\text{mir}}}{\Lambda} = -\frac{m_{3/2}}{2M_{1/2}}. \quad (9)$$

At the mirage scale the anomaly mediation contribution cancel the quantum corrections of the gravity mediation contribution and we get $M_a(M_{\text{mir}}) = M_{1/2}$.

Second we see the trilinear coupling A_{ijk} . The one-loop RG equation is

$$\frac{d}{dt} A_{ijk} = -\frac{1}{4\pi^2} \sum_a (C_i^a + C_j^a + C_k^a) g_a^2 M_a, \quad (10)$$

where C_i^a is the quadratic Casimir coefficient for the field i and $C_i^a = (N^2 - 1)/(2N)$ for a fundamental representation of the gauge group $SU(N)$, $C_i^a = q_i^2$ for the $U(1)$ charge q_i . It is related to the anomalous dimension γ_i as $\gamma_i = 2 \sum_a C_i^a g_a^2$. Then the anomaly mediation

$$A_{ijk}(\mu)|_{\text{anomaly}} = -\frac{1}{16\pi^2} (\gamma_i + \gamma_j + \gamma_k) m_{3/2} \quad (11)$$

and the gravity mediation

$$A_{ijk}(\mu)|_{\text{gravity}} = \tilde{A}_{ijk} - \frac{1}{8\pi^2} (\gamma_i + \gamma_j + \gamma_k) M_{1/2} \ln \frac{\mu}{\Lambda} \quad (12)$$

satisfy the RG equation when they are combined with the $M_a|_{\text{anomaly}}$ and $M_a|_{\text{gravity}}$, respectively. Here \tilde{A}_{ijk} are also the gravity mediation contribution at the cutoff scale. The sum of two contributions $(A_{ijk}|_{\text{anomaly}} + A_{ijk}|_{\text{gravity}}, M_a|_{\text{anomaly}} + M_a|_{\text{gravity}})$ also obeys the same RG equation. As a result,

$$A_{ijk}(\mu) = \tilde{A}_{ijk} - \frac{1}{8\pi^2} (\gamma_i + \gamma_j + \gamma_k) M_{1/2} \ln \frac{\mu}{M_{\text{mir}}}. \quad (13)$$

One can see that the RG evolution of the trilinear coupling also vanishes at M_{mir} .

Lastly we see the scalar mass m_i^2 . The one-loop RG equation is

$$\frac{d}{dt} m_i^2 = -\frac{1}{2\pi^2} \sum_a C_i^a g_a^2 |M_a|^2 + \frac{3}{40\pi^2} g_1^2 Y_i S, \quad (14)$$

where the quantity S is defined as

$$S = \sum_i Y_i m_i^2 = m_{H_u}^2 - m_{H_d}^2 + \text{Tr} [m_{\tilde{Q}}^2 - 2m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 - m_L^2 + m_{\tilde{e}_R}^2]. \quad (15)$$

The scalar mass is generated from the anomaly mediation and the gravity mediation as

$$m_i^2(\mu)|_{\text{anomaly}} = -\frac{1}{32\pi^2} \gamma_i m_{3/2}^2 \quad (16)$$

$$m_i^2(\mu)|_{\text{gravity}} = \tilde{m}_i^2 - \frac{1}{4\pi^2}\gamma_i M_{1/2}^2 \ln \frac{\mu}{\Lambda} - \frac{1}{8\pi^2}\dot{\gamma}_i M_{1/2}^2 \left(\ln \frac{\mu}{\Lambda}\right)^2 + \frac{3}{40\pi^2}Y_i g_1^2 \tilde{S} \ln \frac{\mu}{\Lambda}, \quad (17)$$

where $\dot{\gamma}_i = \frac{d}{dt}\gamma_i$, $\tilde{S} = \sum_i Y_i \tilde{m}_i^2$ and \tilde{m}_i^2 is the mass from the gravity mediation at the cutoff scale. They satisfy the RG equation when they are combined with the $M_a|_{\text{anomaly}}$ and $M_a|_{\text{gravity}}$, respectively. However, the combination $(m_i^2|_{\text{anomaly}} + m_i^2|_{\text{gravity}}, M_a|_{\text{anomaly}} + M_a|_{\text{gravity}})$ does not satisfy the same RG equation. It is not the problem because the scalar mass has interference terms

$$m_i^2(\mu)|_{\text{interference}} = -\frac{1}{8\pi^2}\gamma_i M_{1/2} m_{3/2} - \frac{1}{8\pi^2}\dot{\gamma}_i M_{1/2} m_{3/2} \ln \frac{\mu}{\Lambda} \quad (18)$$

when there are the different SUSY breaking sources. It guarantees the coexistence of the two contributions. After all, the scalar mass under the anomaly mediation and the gravity mediation is

$$m_i^2(\mu) = \tilde{m}_i^2 - \frac{1}{4\pi^2}\gamma_i M_{1/2}^2 \ln \frac{\mu}{M_{\text{mir}}} - \frac{1}{8\pi^2}\dot{\gamma}_i M_{1/2}^2 \left(\ln \frac{\mu}{M_{\text{mir}}}\right)^2 + \frac{3}{40\pi^2}Y_i g_1^2 \tilde{S} \ln \frac{\mu}{\Lambda}. \quad (19)$$

Note that the RG evolution of the scalar mass also cancels at M_{mir} if \tilde{S} vanishes. Hereafter we assume $\tilde{S} = 0$ because it is satisfied in the GUT models where H_u and H_d are unified into a single multiplet, such as $SO(10)$.

We have seen that all the RG evolution effects of gaugino mass, trilinear coupling and scalar mass vanish at the same scale M_{mir} in the small Yukawa case. Therefore we can see that the anomaly mediation effectively lowers the cutoff scale Λ to M_{mir} . In the case of $m_{3/2}/M_{1/2} \sim 60$, the mirage scale is around the TeV scale. Note that the value $m_{3/2}/M_{1/2} \sim 60$ is consistent with the assumptions, $m_{3/2} \sim 100$ TeV, which is for solving the gravitino problem and that the SUSY breaking scale is around 1 TeV.

2.2 Effect of top Yukawa

We have showed that the anomaly mediation cancels the RG evolution of the gravity mediation if there is no Yukawa coupling in the previous subsection. However, the expressions for $m_{H_u}^2$, $m_{\tilde{t}_L}^2$, $m_{\tilde{t}_R}^2$ and A_t should be modified because the top Yukawa coupling has the sizable contribution. Here we consider the case where the bottom and tau Yukawa coupling contributions can be neglected.

Let us see the effect of the top Yukawa coupling in more detail. First, the RG equation of the top Yukawa coupling is

$$\frac{d}{dt}y_t = \frac{1}{16\pi^2}y_t(6y_t^2 - 2\sum_a C_t^a g_a^2) \quad (20)$$

with $C_t^a = C_{\tilde{t}_L}^a + C_{\tilde{t}_R}^a + C_{H_u}^a$. The running top Yukawa coupling is given as

$$y_t^2(\mu) = \frac{y_t^2(\Lambda)E(\mu)}{1 - \frac{3}{4\pi^2}y_t^2(\Lambda)F(\mu)}, \quad (21)$$

where the function $E(\mu)$ and $F(\mu)$ are defined as

$$E(\mu) = \prod_a \left(1 - \frac{b_a}{8\pi^2}g_{\text{GUT}}^2 \ln \frac{\mu}{\Lambda}\right)^{2C_t^a/b_a} \quad (22)$$

$$F(\mu) = \int_{\Lambda}^{\mu} \frac{d\mu'}{\mu'} E(\mu'). \quad (23)$$

The RG equations for the A_t and m_i^2 ($i = \tilde{t}_L, \tilde{t}_R, H_u$) become

$$\frac{d}{dt}A_t = \frac{1}{4\pi^2} \left(3|y_t|^2 A_t - \sum_a C_t^a g_a^2 M_a \right) \quad (24)$$

$$\frac{d}{dt}m_i^2 = -\frac{1}{8\pi^2} \left(k_i|y_t|^2(m_{H_u}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2) + k_i|y_t|^2|A_t|^2 + 4 \sum_a C_i^a g_a^2 |M_a|^2 \right). \quad (25)$$

With the top Yukawa coupling, the A_t , up-type Higgs mass and the stop masses generated by the gravity mediation are given as

$$A_t(\mu) = \tilde{A}_t + 6\rho(\tilde{A}_t - M_{1/2}) - \frac{1}{8\pi^2}(\gamma_{H_u} + \gamma_{t_L} + \gamma_{t_R})M_{1/2} \ln \frac{\mu}{\Lambda}, \quad (26)$$

$$m_i^2(\mu) = \tilde{m}_i^2 - k_i\rho \left[(\tilde{A}_t - M_{1/2})^2(1 + 6\rho) + \tilde{\Sigma}_t - M_{1/2}^2 \right] \\ - \frac{M_{1/2}}{4\pi^2} \left[\gamma_i M_{1/2} + k_i(\tilde{A}_t - M_{1/2})(1 + 6\rho)y_t^2 \right] \ln \frac{\mu}{\Lambda} - \frac{1}{8\pi^2}\dot{\gamma}_i M_{1/2}^2 \left(\ln \frac{\mu}{\Lambda} \right)^2, \quad (27)$$

where $\tilde{\Sigma}_t = \tilde{m}_{H_u}^2 + \tilde{m}_{\tilde{t}_L}^2 + \tilde{m}_{\tilde{t}_R}^2$. The anomalous dimension γ_i is written as

$$\gamma_i = 2 \sum_a C_i^a g_a^2 + k_i y_t^2, \quad (28)$$

where $k_{H_u} = -3, k_{t_L} = -1, k_{t_R} = -2$ and $k_i = 0$ for the other fields. The effect of the top Yukawa coupling is involved in the parameter ρ :

$$\rho(\mu) = \frac{y_t^2(\mu)}{8\pi^2} \frac{F(\mu)}{E(\mu)}. \quad (29)$$

Note that if the function $E(\mu)$ is just a constant, ρ can be estimated as $\rho \sim \ln(\mu/\Lambda)$.

The anomaly mediation changes the expressions (26) and (27) as

$$A_t(\mu) = \tilde{A}_t + 6\rho(\tilde{A}_t - M_{1/2}) - \frac{1}{8\pi^2}(\gamma_{H_u} + \gamma_{t_L} + \gamma_{t_R})M_{1/2} \ln \frac{\mu}{M_{\text{mir}}}, \quad (30)$$

$$m_i^2(\mu) = \tilde{m}_i^2 - k_i\rho \left[(\tilde{A}_t - M_{1/2})^2(1 + 6\rho) + \tilde{\Sigma}_t - M_{1/2}^2 \right] \\ - \frac{M_{1/2}}{4\pi^2} \left[\gamma_i M_{1/2} + k_i(\tilde{A}_t - M_{1/2})(1 + 6\rho)y_t^2 \right] \ln \frac{\mu}{M_{\text{mir}}} - \frac{1}{8\pi^2}\dot{\gamma}_i M_{1/2}^2 \left(\ln \frac{\mu}{M_{\text{mir}}} \right)^2. \quad (31)$$

These analytic formula are given by Ref.[8]. One can see that the cancellation at the mirage scale is spoiled by the top Yukawa contribution. Moreover, the large logarithmic factor appears because $\rho \sim \ln(\mu/\Lambda)$. However, if we impose the special boundary condtions, $\tilde{A}_t = M_{1/2} = \sqrt{\tilde{\Sigma}_t}$, as in the mirage mediation scenario, then the cancellation at the mirage scale can be restored. In the literature[11, 12], the improvement for the tuning has been discussed in the mirage mediation if the mirage scale is around the weak scale.

What happens in the more general case in which the special boundary conditions are not satisfied? We will discuss this subject in the next section.

3 More general cases

In the usual mirage mediation, the special boundary conditions for the gravity contribution to the SUSY breaking parameters are imposed, i.e., the universal sfermion masses to satisfy the flavor changing neutral current (FCNC) constraints, vanishing Higgs masses, and $\tilde{A}_t = M_{1/2} = \sqrt{\tilde{\Sigma}_t}$. In this section, we study more general cases in which the anomaly mediation contribution is sizable.

3.1 Generalization of mirage mediation: Natural SUSY

Before going completely general cases, we discuss the cases in which the cancellation is complete as in the mirage mediation scenario. In these cases, the little hierarchy problem can be quite improved as discussed in the usual mirage mediation. It is obvious that for the cancellation, only the conditions $\tilde{A}_t = M_{1/2} = \sqrt{\tilde{\Sigma}_t}$ are important. For the cancellation, basically no additional condition is required for the other sfermion masses except vanishing \tilde{S} .

As an example, we mention the natural SUSY type boundary conditions[16], in which the sfermion masses m_3 for the third generation **10** of $SU(5)$ are around the TeV scale to stabilize the weak scale, and the other sfermion masses m_0 are taken to be much larger than m_3 to suppress the SUSY contributions to the FCNC processes and CP violation processes. These boundary conditions are consistent with $\tilde{A}_t = M_{1/2} = \sqrt{\tilde{\Sigma}_t}$ and \tilde{S} can vanish. For example, we can adopt the conditions $\tilde{m}_{H_u}^2 = \tilde{m}_{H_d}^2 = 0$ and $\tilde{m}_{\tilde{Q}_3}^2 = \tilde{m}_{\tilde{t}_R}^2 = \tilde{m}_{\tilde{\tau}_R}^2 = \tilde{\Sigma}_t/2$. Similar boundary conditions in the mirage mediation, in which only stop masses are taken to be different from the others, have been discussed in the literature[17], though $\tilde{S} = 0$ is not satisfied in their boundary conditions. We think this possibility interesting because the E_6 GUT with the family symmetry $SU(2)_F$ predicts such natural SUSY type sfermion masses[18].

The most important point is that if the mirage scale is around the SUSY breaking scale, we may directly obtain the signatures of GUT scenarios by observing the sfermion mass spectrum. For example, if the rank of the unification group is larger than the rank of the SM gauge groups, the D -term contribution is non-vanishing generically. We may observe the magnitude of the D -term contribution directly. In the usual arguments, by calculating the RG equations from the SUSY breaking scale to the GUT scale, we can obtain the signatures for the GUT scenarios from the observed sfermion mass spectrum[19]. But in our cases, we do not have to calculate the RG equations or it is sufficient to calculate the RG flow a bit even if we have to. We will return to this point later.

3.2 Upper bound for $m_{3/2}$ from stability conditions

First of all, we explain the gravitino mass range which we would like to study in our scenario. The lower bound of the gravitino mass is about 50 TeV[4], to solve the gravitino problem. Strictly speaking, the lower bound is dependent on the reheating temperature of the inflation. If low reheating temperature is considered, lower $m_{3/2}$ becomes possible. But if thermal leptogenesis is adopted for the baryogenesis, the lower bound of $m_{3/2}$ is not so different from 50 TeV.

If $m_{3/2}$ is so large that the anomaly mediation contribution becomes dominant, the right-handed sleptons must have negative mass square[10]. Therefore, we have upper bound for the gravitino mass. The upper bound for the ratio $m_{3/2}/M_{1/2}$ can be obtained by requiring that the positivity of the stop and stau mass squares at the SUSY breaking scale, or at the GUT scale. From the eq. (19),

the positivity condition for the right-handed stau mass square at μ can be written as

$$\ln \frac{\mu}{M_{\text{mir}}} \leq \frac{10\pi}{33\alpha_1(\mu)} \left[\sqrt{1 + 5.5 \frac{\tilde{m}_{\tilde{\tau}_R}^2}{M_{1/2}^2}} - 1 \right]. \quad (32)$$

Since $\ln \frac{\mu}{M_{\text{mir}}} = \ln \frac{\mu}{\Lambda} + \frac{m_{3/2}}{2M_{1/2}}$, this gives the upper bound for $m_{3/2}$. If we take $\tilde{m}_{\tilde{\tau}_R} = M_{1/2}$, the upper bound for the gravitino mass becomes $222M_{1/2}$ for $\mu = 1$ TeV and $76M_{1/2}$ for $\mu = \Lambda_G$. For the stop masses, numerical upper bounds are given in Fig. 1 for $M_{1/2} = 2$ TeV and $\tilde{A}_t = -2000, 2000, 4000$ TeV. In the calculation, we assume that $\tilde{m}_{\tilde{t}_R} = \tilde{m}_{\tilde{t}_L} = \tilde{m}_{\tilde{\tau}_R} \equiv \tilde{m}$ and $\tilde{m}_{H_u} = 0$. All sfermion mass squares must be positive at least at the SUSY breaking scale. For this minimal requirement, roughly $m_{3/2} < 100M_{1/2}$ if $\tilde{m} < M_{1/2}$, and $m_{3/2} < 500M_{1/2}$ if $\tilde{m} < 2M_{1/2}$. If the positivity at the GUT scale is required (though this is not necessary for the consistency of the theory), $m_{3/2} < 200M_{1/2}$ when $\tilde{m} < 2M_{1/2}$. In numerical calculations in this paper, we take $m_t(\text{pole}) = 173.07$ GeV and the unified gauge coupling $g_{\text{GUT}}^2 = 0.48$. We are interesting in the region $30 < m_{3/2}/M_{1/2} < 200$ in this paper.

Note that under the special boundary conditions $M_{1/2} = \tilde{A}_t = \sqrt{2}\tilde{m}_{\tilde{t}}$ in the mirage mediation the some sfermion mass squares become negative at the GUT scale as seen in the figure. However, in the general boundary conditions, the positivity at the GUT scale can be satisfied.

3.3 Improvement in general cases

In this subsection, we show that even in the general cases, the fine-tuning can be improved by using the numerical calculation.

First, we explain the improvement in the mirage mediation. Let us evaluate the quantum correction for the Higgs mass $m_{H_u}^2(\mu = 1\text{TeV})$ from the eq.(27) obtained in the gravity mediation. We express the quantum correction $\Delta m_{H_u}^2 = m_{H_u}^2 - \tilde{m}_{H_u}^2$ as

$$\Delta m_{H_u}^2(1\text{TeV}) = c_0 M_{1/2}^2 + c_1 \tilde{\Sigma}_t + c_2 \tilde{A}_t^2 + c_3 \tilde{A}_t M_{1/2}, \quad (33)$$

where constants c_i are numerically calculated as

$$c_0 = -1.601, \quad c_1 = -0.396, \quad c_2 = -0.082, \quad c_3 = -0.260. \quad (34)$$

If we set $M_{1/2} = \tilde{A}_t = \sqrt{\tilde{\Sigma}_t}$, we obtain $\Delta m_{H_u}^2 = -2.34M_{1/2}^2$. In order to obtain the quantum correction for the Higgs mass in the mirage mediation, we reevaluate c_i under the anomaly mediation from the eq. (31). If we set $m_{3/2}/M_{1/2} = 60.0$, we obtain

$$c_0 = 0.291, \quad c_1 = -0.396, \quad c_2 = -0.082, \quad c_3 = 0.156. \quad (35)$$

If we take the boundary conditions in the mirage mediation as $M_{1/2} = \tilde{A}_t = \sqrt{\tilde{\Sigma}_t}$, we obtain $\Delta m_{H_u}^2 = -0.031M_{1/2}^2$. These calculations show that the mirage mediation has more than one order less tuning is required than the gravity mediation without the anomaly mediation. The essential points for this improvement are that the coefficients c_i become small and the cancellation happens because of the different signatures of c_i .

These points for the improvement are also applicable to the more general cases. Therefore, it is obvious that even for the general cases, some improvement for the tuning can be expected at least when the ratio $m_{3/2}/M_{1/2} = 60$. Is this improvement realized only in this special value for the ratio? Note that c_1 and c_2 do not change by including the anomaly mediation. On the other hand,

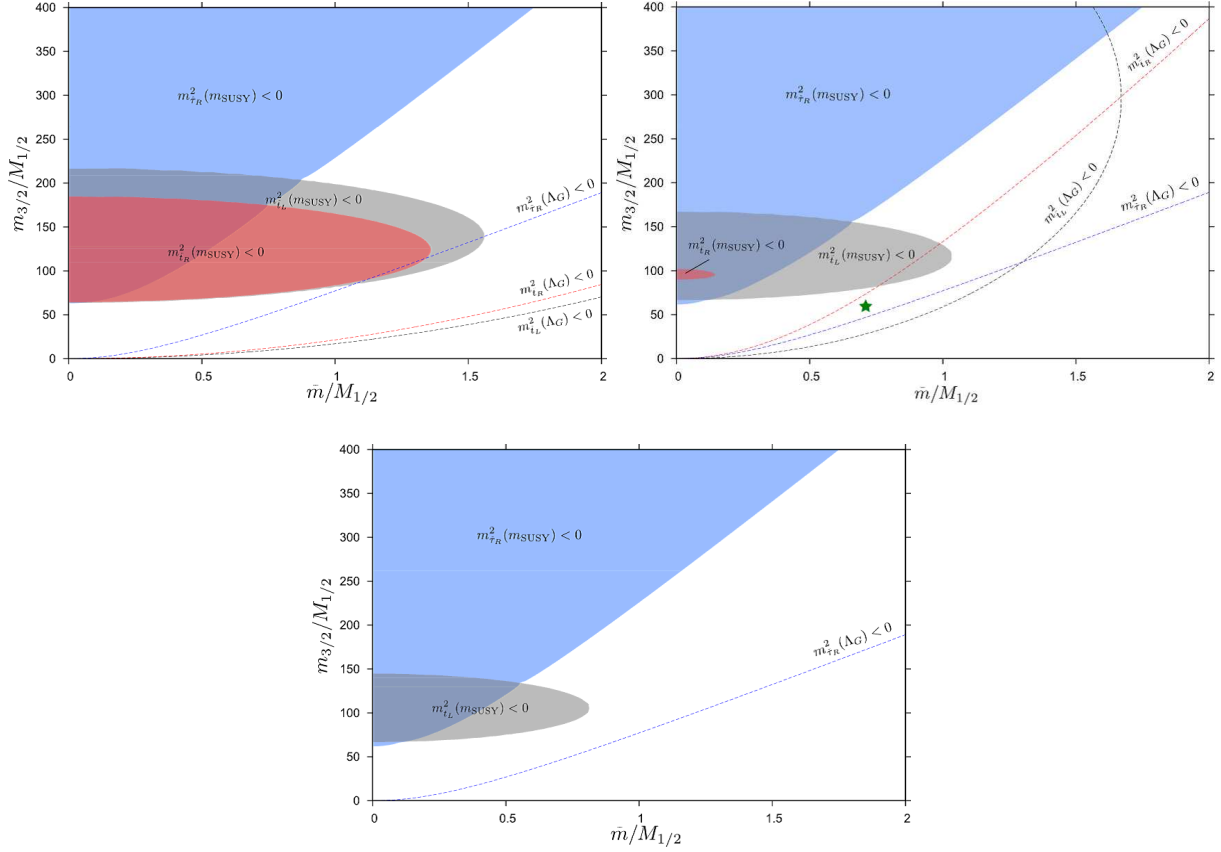


Figure 1: Allowed region for the stability conditions at 1 TeV and at the GUT scale Λ_G in $(\tilde{m}/M_{1/2}, m_{3/2}/M_{1/2})$ plain, where $\tilde{m} = \tilde{m}_{\tilde{t}_L} = \tilde{m}_{\tilde{t}_R} = \tilde{m}_{\tilde{\tau}_R}$. The shaded region is forbidden by the stability conditions at 1 TeV, and the upper side of the dotted line is the region where the mass square is negative at the GUT scale. The upper left figure is for $\tilde{A}_t = -2000$ GeV, the upper right figure is for $\tilde{A}_t = 2000$ GeV, and the lower figure is for $\tilde{A}_t = 4000$ GeV. The mirage point is dotted by star symbol. Note that in the mirage point, the GUT scale stability cannot be satisfied.

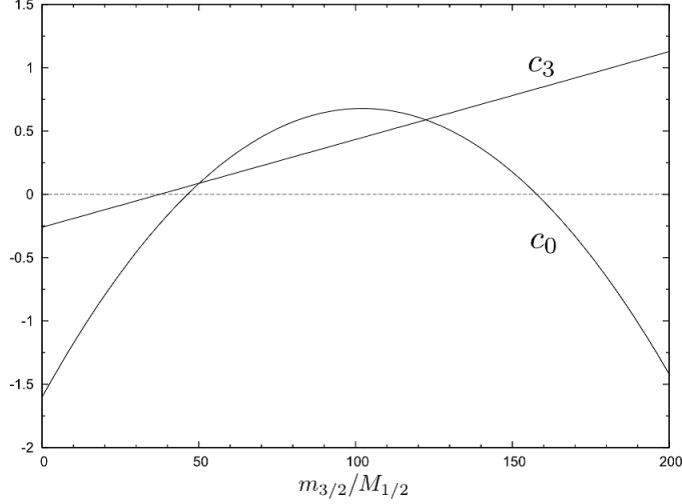


Figure 2: Values of c_0 and c_3 in eq. (33) versus $m_{3/2}/M_{1/2}$.

c_0 and c_3 depend on M_{mir} , namely, $m_{2/3}/M_{1/2}$. Figure 2 shows this dependence. One can see that the absolute values of c_0 and c_3 are reduced by the anomaly mediation with wide range of value of $m_{2/3}/M_{1/2}$ among the range we are interested in. Actually, if $29 < m_{2/3}/M_{1/2} < 73$, the condition $|c_i| < 0.5$ is satisfied for $i = 0, 1, 2, 3$. Therefore, we conclude that even in the general cases, some improvements for the tuning problem are expected in our scenario.

Here we numerically check whether the quantum correction of the Higgs mass $\Delta m_{H_u}^2$ can be small. In our scenario $\Delta m_{H_u}^2$ depends on four parameters: $M_{1/2}$, \tilde{A}_t , $\tilde{\Sigma}_t$ and M_{mir} . Hereafter we use $m_{3/2}/M_{1/2}$ instead of M_{mir} . Figure 3 shows $m_h^2/|2\Delta m_{H_u}^2|$ ($\mu = m_{\text{SUSY}}$) in $(\tilde{A}_t, M_{1/2})$ plain with $\sqrt{\tilde{\Sigma}_t} = 2$ TeV which corresponds to $\tilde{m}_{\tilde{t}_L} = \tilde{m}_{\tilde{t}_R} = \sqrt{2}$ TeV. Therefore, roughly, $m_{\text{SUSY}} \sim \sqrt{2}$ TeV. The dark gray, gray and light gray regions are correspond to $(m_h^2/2)/|\Delta m_{H_u}^2| > 0.1, 0.02$ and 0.01 , respectively, where m_h is the Higgs mass measured at the LHC as $m_h \sim 125$ GeV. One can see that the tuning weaker than one percent is realized in a wide range of parameters. (Strictly speaking, we have to address how strong tuning is required for model parameters to be included in these areas. From these figures, we can see that $O(1\%)$ tuning is required in this scenario. Since this value is better than $O(0.1\%)$ tuning for the usual minimal SUGRA boundary conditions, we can conclude that the tuning problem becomes less severe.) Note that the amount of tuning for realizing small $\Delta m_{H_u}^2$ increases as $M_{1/2}$ becomes large as seen in Fig. 4. Therefore the masses of the gauginos should not be much larger than TeV scale if we expect not so large tuning with model parameters for getting small $\Delta m_{H_u}^2$. One more important feature in general cases is that A_t can be as large as $\sqrt{6}m_{\tilde{t}}$, which results in the maximal Higgs mass. This is an advantage in the general cases. One can check from figure 3 that $|\Delta m_{H_u}^2|$ is not influenced much by the value of \tilde{A}_t . On the other hand, the large A_t is important to obtain heavier Higgs. In the figure 3, we calculate the lightest Higgs mass by using the program FeynHiggs-2.9.5 [20] under the additional assumptions which are adopted in the Ref. [12]. Namely we assume that $\tilde{m}_{\tilde{t}_L}^2 = \tilde{m}_{\tilde{t}_R}^2 = \tilde{\Sigma}_t/2$ and the parameters μ , $\tan \beta$ and the mass of the CP odd Higgs m_A are fixed by hand at the SUSY breaking scale. (The latter assumption can be adopted if the unknown GUT threshold corrections to the Higgs mass parameters are taken into account as noted in the Ref. [12].) Therefore we can realize the 125 GeV Higgs with small $m_{\tilde{t}}$ by setting $A_t/m_{\tilde{t}} \simeq \sqrt{6}$. Actually when $50 \leq m_{3/2}/M_{1/2} \leq 90$, 125 GeV Higgs mass can be realized

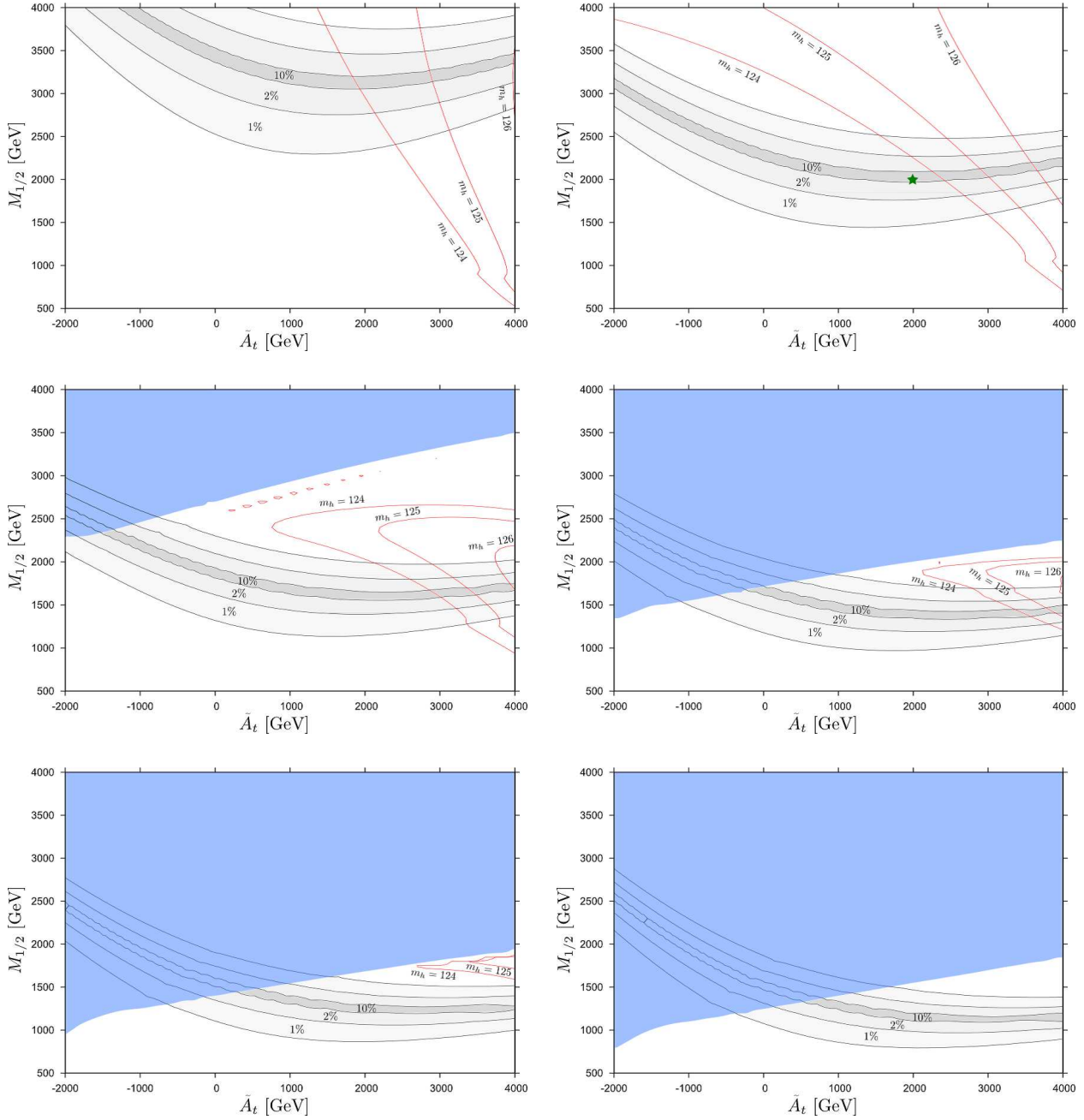


Figure 3: $m_h^2/|2\Delta m_{Hu}^2(m_{\text{SUSY}})|$ in $(\tilde{A}_t, M_{1/2})$ plane for $m_{3/2}/M_{1/2} = 50$ (upper-left), 60 (upper-right), 70 (middle-left), 80 (middle-right), 90 (lower-left), 100 (lower-right). We take $\sqrt{\tilde{\Sigma}_t} = 2$ TeV. The shaded region is forbidden by the stability conditions at m_{SUSY} . For reference, the Higgs mass, which is calculated by taking $\mu = m_A = 500$ GeV and $\tan\beta = 10$, is shown as lines for 124 GeV, 125 GeV, and 126 GeV. The mirage point is dotted by the star symbol. The stability condition $m_{\tilde{\tau}_R}^2 \geq 0$ at the GUT scale leads to the upper bound for $M_{1/2}$ as 1.91 TeV, 1.69 TeV, 1.51 TeV, 1.38 TeV, 1.26 TeV, and 1.17 TeV for $m_{3/2}/M_{1/2}=50, 60, 70, 80, 90$, and 100, respectively. For large \tilde{A}_t , all sfermion mass squares can be positive till the GUT scale if this condition is satisfied.

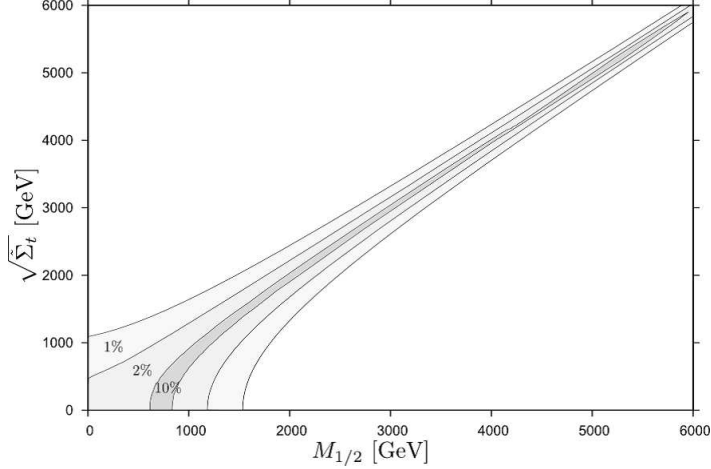


Figure 4: $m_h^2/|2\Delta m_{Hu}^2(m_{\text{SUSY}})|$ in $(M_{1/2}, \sqrt{\tilde{\Sigma}_t})$ plain. We take $\tilde{A}_t = 2$ TeV and $m_{3/2}/M_{1/2} = 60$.

with reasonable value for \tilde{A}_t as seen in Fig. 3. On the other hand, if $m_{3/2}/M_{1/2}$ is larger than 100, no line for 125 GeV Higgs appears because the stop masses are too small when $\tilde{m}_{\tilde{t}} = \sqrt{2}$ TeV.

This interesting feature can be understood also from the numerical formula (33), which is rewritten as

$$\Delta m_{Hu}^2(1\text{TeV}) = \bar{c}_0 M_{1/2}^2 + c_1 \tilde{\Sigma}_t + c_2 \left(\tilde{A}_t + \frac{c_3}{2c_2} M_{1/2} \right)^2, \quad (36)$$

where $\bar{c}_0 \equiv c_0 - c_3^2/(4c_2)$. Since without the anomaly mediation contribution, all parameters \bar{c}_0 , c_1 , and c_2 are negative, Δm_{Hu}^2 cannot be zero nor small and therefore tuning becomes worse. However, if anomaly mediation contribution is sizable, \bar{c}_0 can be positive and therefore, Δm_{Hu}^2 can vanish. How large anomaly mediation contribution is needed for positive \bar{c}_0 ? Numerically, $m_{3/2} \geq 47M_{1/2}$ is needed. What is important here is that Δm_{Hu}^2 is dependent on \tilde{A}_t quite mildly when $\tilde{A}_t \sim -c_3 M_{1/2}/(2c_2)$, which is derived from $\partial \Delta m_{Hu}^2 / \partial \tilde{A}_t = 0$. The scale of $M_{1/2}$ for vanishing Δm_{Hu}^2 can be determined by cancellation condition for the first two terms in eq. (36) as $M_{1/2} \sim \sqrt{-c_1 \tilde{\Sigma}_t / \bar{c}_0} = \sqrt{-2c_1 / \bar{c}_0} \tilde{m}_{\tilde{t}}$. Note that the ratio $\tilde{m}_{\tilde{t}}/M_{1/2} = \sqrt{-\bar{c}_0/(2c_1)}$ is important in deriving the stability conditions as in Fig. 1. These values for various $m_{3/2}/M_{1/2}$ are found in Table 1.

From both relations $\tilde{A}_t \sim -c_3 M_{1/2}/(2c_2)$ and $M_{1/2} \sim \sqrt{-c_1 \tilde{\Sigma}_t / \bar{c}_0} = \sqrt{-2c_1 / \bar{c}_0} \tilde{m}_{\tilde{t}}$, an interesting relation $\tilde{A}_t / \tilde{m}_{\tilde{t}} = \sqrt{-c_1 c_3^2 / (2\bar{c}_0 c_2^2)}$ is obtained. Surprisingly, in very wide range of $m_{3/2}/M_{1/2}$, the coefficient $\sqrt{-c_1 c_3^2 / (2\bar{c}_0 c_2^2)}$ is around 2 as in Table 1. This means that the interesting feature, that Δm_{Hu}^2 has quite mild dependence on \tilde{A}_t around $\tilde{A}_t \sim 2$, and therefore we can obtain 125 GeV Higgs easier by taking large \tilde{A}_t , is generally realized in this scenario.

The lower bound for the ratio $m_{3/2}/M_{1/2}$ which realizes $\Delta m_{Hu}^2 = 0$ is also shown in Fig. 5 in which $\tilde{A}_t = \sqrt{\tilde{\Sigma}_t} = 2$ TeV. This lower bound is consistent with the above arguments from the numerical formula (36). Even the upper bound for the ratio $m_{3/2}/M_{1/2}$ is seen in Fig. 5. The upper bound becomes lower than the value discussed in the above, because \tilde{A}_t is fixed to be 2 TeV in the numerical calculation in Fig. 5. Interestingly the lower bound for $M_{1/2}$ is seen in the figure.

$\frac{m_{3/2}}{M_{1/2}}$	10	30	50	60	70	80	90	100	120	150	200
c_0	-1.177	-0.458	0.085	0.291	0.453	0.572	0.646	0.677	0.608	0.176	-1.418
c_3	-0.191	-0.052	0.087	0.156	0.225	0.295	0.364	0.433	0.572	0.780	1.127
\bar{c}_0	-1.066	-0.450	0.108	0.365	0.607	0.837	1.050	1.249	1.606	2.031	2.454
$\sqrt{-\frac{\bar{c}_0}{2c_1}}$	-	-	0.369	0.679	0.876	1.023	1.152	1.256	1.425	1.603	1.761
$-\frac{c_3}{2c_2}$	-1.165	-0.317	0.530	0.951	1.372	1.799	2.220	2.640	3.488	4.756	6.872
$\sqrt{-\frac{c_1 c_3^2}{2\bar{c}_0 c_2^2}}$	-	-	1.435	1.401	1.567	1.750	1.927	2.101	2.449	2.968	3.903

Table 1: Coefficients c_0 , c_3 , etc. for various $m_{3/2}/M_{1/2}$. $\Delta m_{H_u}^2 = 0$ and $\partial \Delta m_{H_u}^2 / \partial \tilde{A}_t = 0$ lead to $\sqrt{-\bar{c}_0/(2c_1)} = \tilde{m}_{\tilde{t}}/M_{1/2}$, $-c_3/(2c_2) = \tilde{A}_t/M_{1/2}$, respectively, and therefore $\sqrt{-c_1 c_3^2/(2\bar{c}_0 c_2^2)} = \tilde{A}_t/\tilde{m}_{\tilde{t}}$.

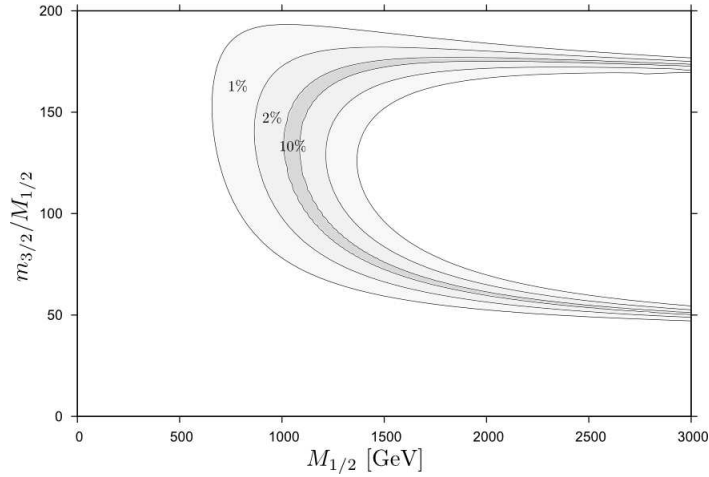


Figure 5: $m_h^2/|2\Delta m_{H_u}^2(m_{\text{SUSY}})|$ in $(M_{1/2}, m_{3/2}/M_{1/2})$ plain. We take $\tilde{A}_t = \sqrt{\tilde{\Sigma}_t} = 2$ TeV.

3.4 Strategy for testing GUT in general cases

In this subsection, we discuss how to obtain the signatures for GUT scenarios from the mass spectrum of the SUSY particles, which is assumed to be observed by experiments here, in general cases. As noted in the previous section, the top Yukawa contribution spoils the cancellation between the RG contribution and anomaly mediation contribution for the up-type Higgs mass and stop masses at M_{mir} . However, for the other sfermion masses and the gaugino masses, the cancellation at M_{mir} is still valid. Therefore, from the mass spectrum of the gauginos, we can obtain the mirage scale M_{mir} by calculating the RG equations for gaugino masses. Once the mirage scale is known, we can obtain the gravity contribution to the masses of the sfermions other than two stops by calculating the RG equations from the SUSY breaking scale to the mirage scale. It is beyond the scope of this paper how to test a concrete GUT scenario by this strategy. We will study this subject in future.

4 Summary and discussion

We have shown that if we require that $m_{3/2} \sim O(100\text{TeV})$ for solving the gravitino problem and the other SUSY breaking parameters are $O(1\text{TeV})$ for the naturalness, the little hierarchy problem becomes less severe. The essential point is that in such a situation, the anomaly mediation contribution becomes sizable, which can generically lower the messenger scale of the gravity mediation effectively.

If the Yukawa coupling is negligible, all the RG evolutions of gaugino mass, the scalar mass and trilinear coupling are canceled at the same scale M_{mir} by the anomaly mediation contribution. However, the Yukawa contribution breaks the complete cancellation at M_{mir} for the scalar and the trilinear coupling. In practice the large top Yukawa coupling spoils the cancellation at M_{mir} for the stop masses, up-type Higgs mass, and A_t . One possibility for vanishing the top Yukawa contribution is that the special boundary conditions are adopted such as the mirage mediation. This special boundary conditions are applied only for the stop masses and up-type Higgs masses, and therefore, we have no constraints for the other sfermion masses. First, we discussed the generalization of the mirage mediation. It is interesting that the natural SUSY mass spectrum is consistent with the mirage type boundary conditions. Second, we have considered another possibility in which we do not have special boundary conditions for the gravity contributions. We have showed that even in such general cases, the tuning is improved in a wide range of parameter spaces we are interested in. An attractive feature of this scenario is that it has the flexibility of the mass parameters at the cutoff scale because we need not exactly cancel the top Yukawa contribution. We can get large values like $A_t/m_{\tilde{t}} \simeq \sqrt{6}$, which is important for realizing 125 GeV Higgs with smaller $m_{\tilde{t}}$.

One of the disadvantage of the gravity mediation is that the universality of the sfermion masses, which are important in solving the SUSY FCNC problem, is not guaranteed generically. One interesting possibility is to introduce flavor symmetry to realize the universality. One of the most interesting symmetries is $E_6 \times SU(2)_F$ which realize the modified universality in which the third generation **10** of $SU(5)$ can have different mass m_3 than the other sfermion mass m_0 . If we take $m_0 \gg m_3 \sim 1\text{ TeV}$, this is nothing but the natural SUSY type SUSY breaking parameters.

Our new scenario has the several interesting features. First, the mirage scale M_{mir} , where the quantum corrections for the gaugino and the scalar masses which does not couple with top vanish, need not to be just the TeV scale. The scale M_{mir} can be smaller than the weak scale, so long as the correction of the Higgs mass is not so large. Then the lightest gaugino may be the gluino unlike the TeV-scale mirage mediation.

Second, this model predicts that the mass difference of two stop masses is around the weak scale, even if these masses are around the TeV scale. Suppose two stop masses from the gravity mediation unifies at the cutoff scale

$$\tilde{m}_{\tilde{t}_L}^2 = \tilde{m}_{\tilde{t}_R}^2. \quad (37)$$

This is expected from the GUT models such as $SU(5)$. The top Yukawa contribution splits these masses even at the mirage scale M_{mir} . However, these masses nearly degenerate if $\Delta m_{H_u}^2$ is small because the relation

$$\Delta m_{\tilde{t}_L}^2 - \Delta m_{\tilde{t}_R}^2 = -\frac{1}{3}\Delta m_{H_u}^2 + \frac{M_{1/2}^2}{\pi}(-2\alpha_2 + \frac{2}{5}\alpha_1) \ln \frac{\mu}{M_{\text{mir}}} + \frac{M_{1/2}^2}{8\pi^2}(-4\alpha_2^2 + \frac{132}{25}\alpha_1^2) \left(\ln \frac{\mu}{M_{\text{mir}}} \right)^2 \quad (38)$$

can be found. Note the QCD and top Yukawa contributions cancel between two stop masses, therefore the mass difference is approximately proportional to the correction of the Higgs mass.

If the naturalness is required, the Higgsino mass μ must not be much larger than the weak scale. Therefore, the lightest SUSY particle (LSP) can be expected to be the Higgsino. If it is additionally required that the thermally produced Higgsino abundance is consistent with the observed abundance of the dark matter, we can obtain further constraints on SUSY parameters. We do not discuss this direction in detail.

One of the most important features in our scenario is that if the mirage scale is around the SUSY breaking scale, the signatures of the GUT scenarios can be observed directly by observing the mass spectrum of SUSY particles. It is difficult to reach the GUT scale directly by experiments while the SUSY GUT is the most promising candidates as the physics beyond the SM. Therefore, it becomes quite important that future experiments can observe the signature of the SUSY GUT, for example, through the D -term contributions to the sfermion masses.

Acknowledgments

K.T. is supported by Grants-in-Aid for JSPS fellows. N.M. is supported in part by Grants-in-Aid for Scientific Research from MEXT of Japan.

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